Math 1A: Calculus **Instructor: Alexander Paulin**

Handout: Practice with the Definition of a Derivative

Discussions 201, 203 // 2018-09-21

Let f be a function. Recall that if the limit

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

exists, then we say it is *the derivative of f at a*, and denote it by $f'(a)$.

Problem 1. Let $f(x) = (x-2)^2$. Compute $f(5)$ and $f'(5)$.

Problem 2. Let $g(x) = \sqrt{x} + 2$. Compute $g(9)$ and $g'(9)$.

Compare your answers to the preceding problems. Draw a picture and see if you understand the connection between them!

Problem 3. Recall from lecture a while ago that the number e has the property that the tangent line to the graph of $y = e^x$ at the point $(0,1)$ has slope exactly 1.

In other words, with $f(x) = e^x$, it is true that $f'(0) = 1$.

Assuming this fact, compute $f'(a)$ in terms of a, for any number a.

Problem 4. Suppose f is some function and g is defined as $g(x) = f(cx)$ for some nonzero constant c. (Do you remember how the graph of g compares to the graph of f in such a situation?)

Let a be any number. Find a way to relate the quantity $g'(a)$ to the quantity $f'(ca)$ (provided both quantities exist). This is a special case of the so-called *chain rule* that we will cover later.

Problem 5. Let $g(x) = b^x$ for some base $b > 0$ which is not equal to 1. Compute $g'(a)$ for arbitrary a. Hint: see if you can make use of Problems 3 and 4. How are $g(x) = b^x$ and $f(x) = e^x$ related?

Remark. When people actually want to compute derivatives, they typically don't use the definition to do so! It's the same thing as with the whole epsilon-delta limit business—we're showing you these things so that you understand where limits and derivatives *come from*. Soon we will develop more sophisticated ways of computing derivatives in practice ("derivative laws" so to speak, just as how we had "limit laws").